Political Science 209 - Fall 2018

Linear Regression

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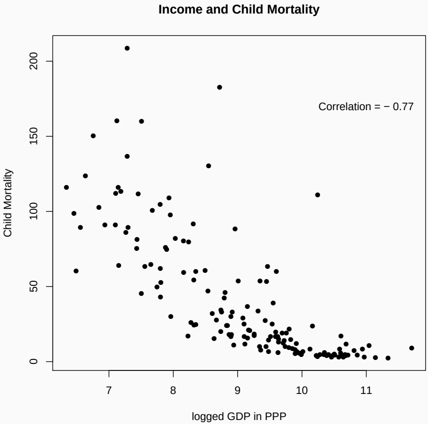
12th October 2018

Recall Correlation & Scatterplot



What is the correlation?

Recall the definition of correlation

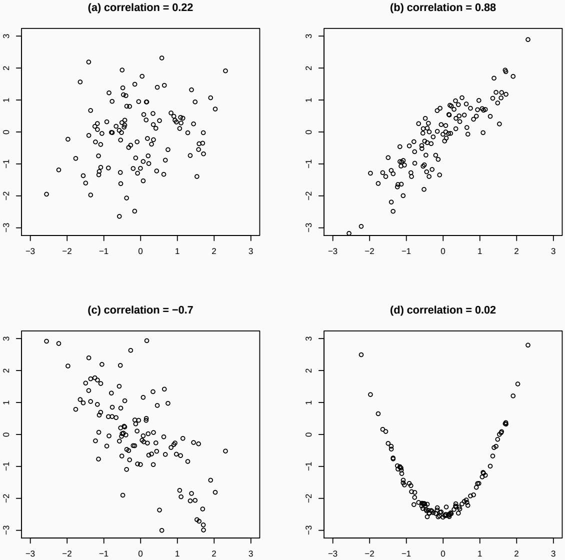
Correlation (x,y) $= \frac{1}{N} \sum_{i=1}^{N}$ z-score of $x_i \times$ z-score of y_i

Correlation (x,y) $= \frac{1}{N} \sum_{i=1}^{N} \frac{x_i - \bar{x}}{sd_x} \times \frac{y_i - \bar{y}}{sd_y}$

Correlations & Scatterplots/Data points

1. positive correlation $\rightsquigarrow$ upward slope
2. negative correlation $\rightsquigarrow$ downward slope
3. high correlation $\rightsquigarrow$ tighter, close to a line
4. correlation cannot capture nonlinear relationship

Correlations & Scatterplots/Data points



Moving from Correlation to Linear Regression

Preview:

- linear regression allows us to create predictions
- linear regression specifies direction of relationship
- linear regression allows us to examine more than two variables at the same time (*statistical control*)

Linear Regression

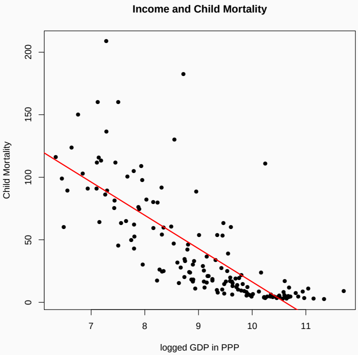
- regression has one dependent (y) and *for now* one independent (x) variable
- regression is a statistical method to estimate the linear relationship between variables

- goal of regression is to approximate the (linear) relationship between X and Y as best as possible

Linear Regression

- goal of regression is to approximate the (linear) relationship between X and Y as best as possible
- regression is the mathematical model to draw best fitting line through cloud of points

Linear Regression



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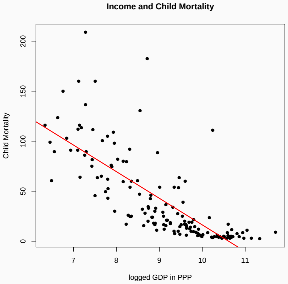
Linear Regression



- regression line is an estimate of the (for now bivariate) relationship between x and y
- for each x we have a prediction of y: what would we expect y to be given the value of x?

What is the equation of a line?

Equation of a line?

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Equation of a line? $y = mx + b$

$\rightarrow$ b? m?

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regression equation:

$Y = \alpha + \beta X + \epsilon$

$\rightarrow \alpha?\ \beta?\ \epsilon?$

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regression equation:

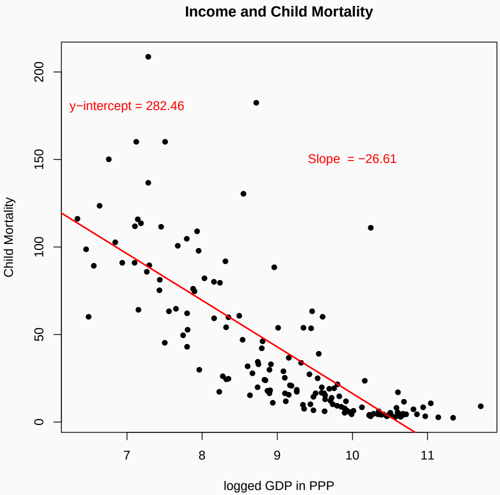
$Y = alpha + \beta X + \epsilon$

$\alpha \rightarrow$ y-intercept

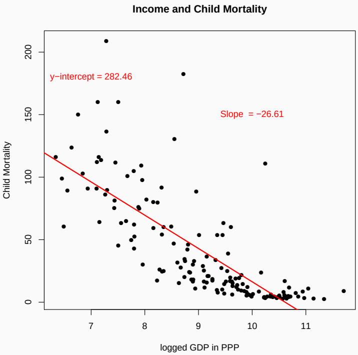
$\beta \rightarrow$ slope

$\epsilon \rightarrow$ error

Regression equation



Regression equation



$$Y = 282.46 + -26.61X + \epsilon$$

Regression equation

Model:

$$Y = \underbrace{\alpha}_{\text{intercept}} + \underbrace{\beta}_{\text{slope}} X + \underbrace{\epsilon}_{\text{error term}}$$

- Y: dependent/outcome/response variable
- X: independent/explanatory variable, predictor
- (α, β): coefficients (parameters of the model)
- ϵ: unobserved error/disturbance term (mean zero)

Regression: Interpretation of the Parameters:

$$Y = \underbrace{\alpha}_{\text{intercept}} + \underbrace{\beta}_{\text{slope}} X + \underbrace{\epsilon}_{\text{error term}}$$

- $\alpha + \beta X$: average of Y at the given the value of X
- α: the value of Y when X is zero
- β: increase in Y associated with one unit increase in X

Regression equation

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- our regression line is an estimate, based on the collected data

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- estimates are denoted with little hats: $\hat{\beta}$, $\hat{\alpha}$
- $(\hat{\alpha}, \hat{\beta})$: estimated coefficients

- we can use $(\hat{\alpha}, \hat{\beta}, X)$ to create *predicted values* of y
- $\widehat{Y} = \hat{\alpha} + \hat{\beta}x$: predicted/fitted value

Regression equation

How far off is our line? How do we know?

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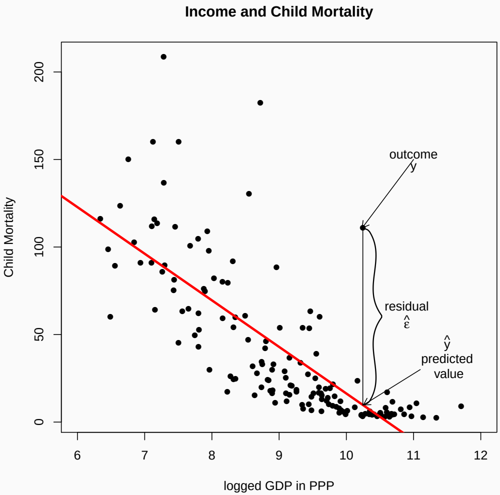
How far off is our line? How do we know?

$\hat{\epsilon} =$ true $Y - \widehat{Y}$: residuals/error

$\hat{\epsilon}$'s are an estimate of how good/bad our line approximates the relationship

Regression



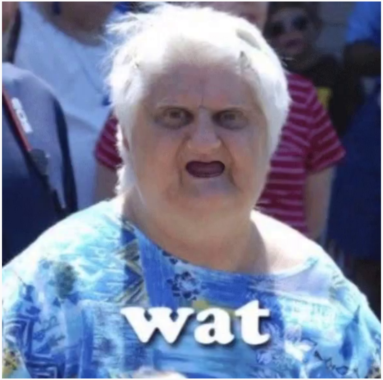
Income and Child Mortality
Child Mortality
logged GDP in PPP
outcome
y
residual
$\hat{\varepsilon}$
$\hat{y}$
predicted
value
0
50
100
150
200
6
7
8
9
10
11
12

Regression

- (α, β) are estimated from the data
- How do we find α, β?

Regression: How do we find α, β?

We minimize the sum of the squared residuals



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We minimize the *sum of the squared residuals (SSR)*

$$\text{SSR} = \sum_{i=1}^{n} \hat{\epsilon}_i^2$$

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This also minimizes the root mean squared error: $\text{RMSE} = \sqrt{\frac{1}{n}\text{SSR}}$

Regression by Hand

$$
\begin{aligned}
\hat{\alpha} &= \bar{Y} - \hat{\beta}\bar{X} \\
\hat{\beta} &= \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})(X_i - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2}
\end{aligned}
$$

OR:

Regression by Hand

$$\hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})(X_i - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

OR:

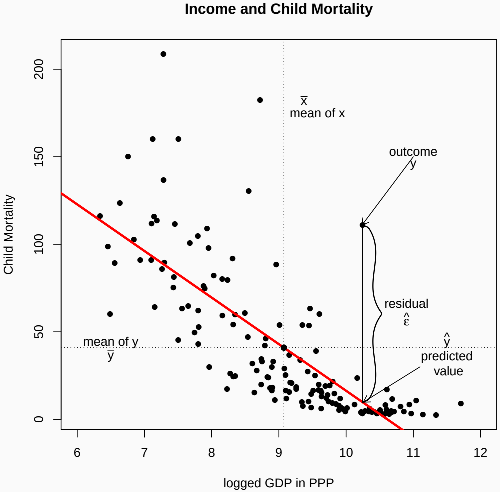
$$\hat{\beta} = \text{correlation of } X \text{ and } Y \times \frac{\text{standard deviation of } Y}{\text{standard deviation of } X}$$

Regression by Hand

Regression line always goes through the point of averages $(\hat{X}, \hat{Y})$

$$\widehat{Y} \;=\; (\overline{Y} - \hat{\beta}\overline{X}) + \hat{\beta}\overline{X} \;=\; \overline{Y}$$

Regression always goes through point of averages



Regression NOT by Hand

Enough math!

Fitting/estimating a regression in *R*:

```
lm(dependent ~ independent, data = data_object)
```

Regression NOT by Hand

Fitting/estimating a regression in *R*:

```
data <- read.csv("bivariate_data.csv")
data <- subset(data, Year ==2010)
result <- lm(Child.Mortality ~ log(GDP) , data = data)
summary(result)
```

Regression NOT by Hand

```r
result <- lm(Child.Mortality ~ log(GDP) , data = data)
coef(result) ### coefficients
```

```
(Intercept)    log(GDP)
  282.45870   -26.61347
```

R-output:

(Intercept): α

log(GDP): β

Model Fit

How well does our regression line fit the data?

How well does the model predict the outcome?

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How well does the model predict the outcome?

R^2 or *coefficient of determination*:

$$R^2 \;=\; 1 - \frac{\text{SSR}}{\text{Total sum of squares (TSS)}} \;=\; 1 - \frac{\sum_{i=1}^{n} \hat{\epsilon}_i^2}{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}$$

Model Fit

$$R^2 \;=\; 1 - \frac{\text{SSR}}{\text{Total sum of squares (TSS)}} \;=\; 1 - \frac{\sum_{i=1}^{n} \hat{\epsilon}_i^2}{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}$$

R^2 is also defined as the *explained variance* in Y

How much of the deviation of Y from the average is explained by X?

Model Fit

```
result <- lm(Child.Mortality ~ log(GDP) , data = data)
summary(result)
```

```

Call:
lm(formula = Child.Mortality ~ log(GDP), data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-49.455 -15.418  -4.161  10.847 132.136

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  282.459     16.569   17.05   <2e-16 ***
log(GDP)     -26.613      1.809  -14.71   <2e-16 ***
---
codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 27.57 on 150 degrees of freedom
Multiple R-squared:  0.5906,Adjusted R-squared:  0.5878
F-statistic: 216.4 on 1 and 150 DF,  p-value: < 2.2e-16
```