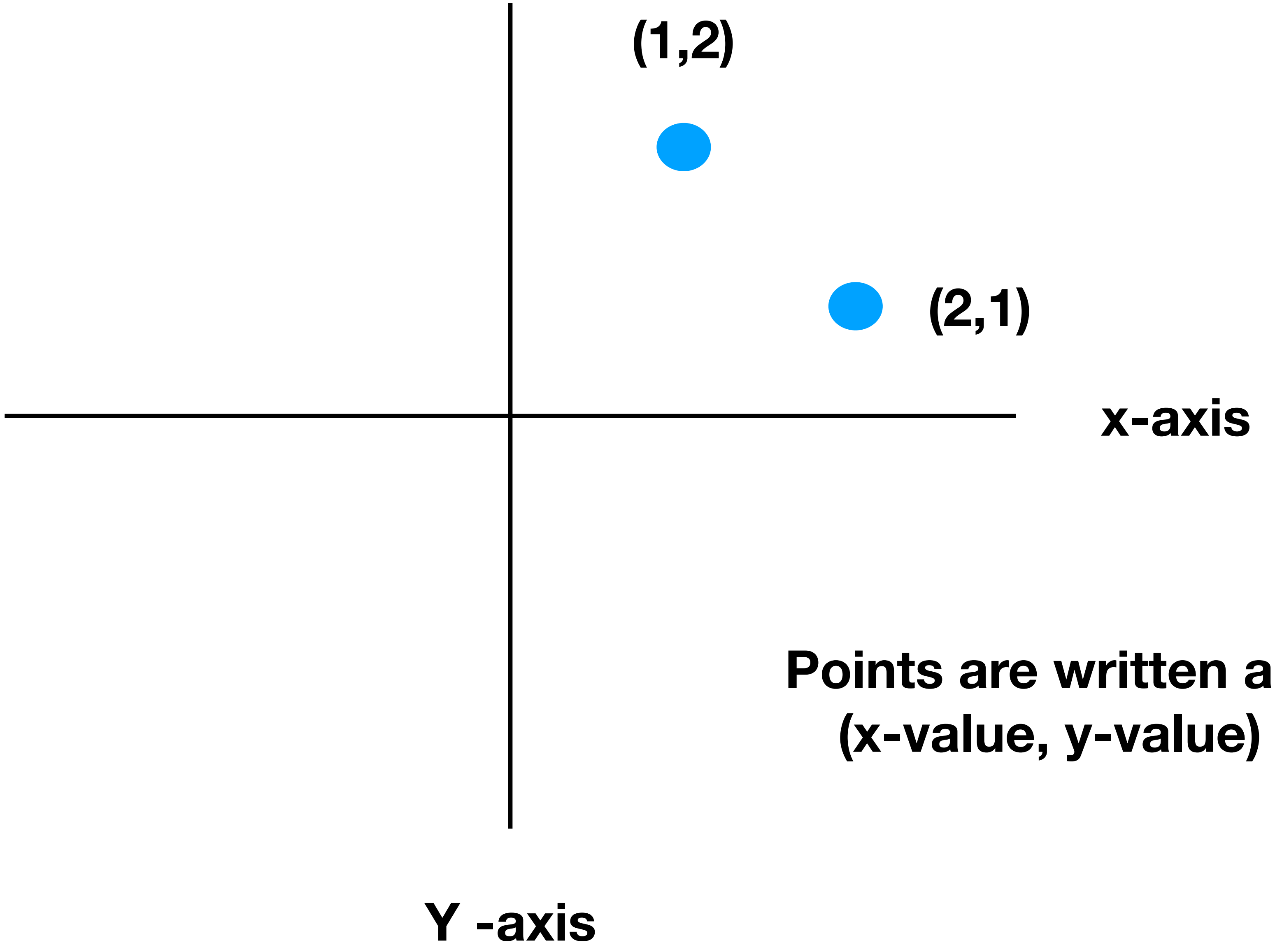
Correlation and Scatter Plots

how to describe the relationship between two variables

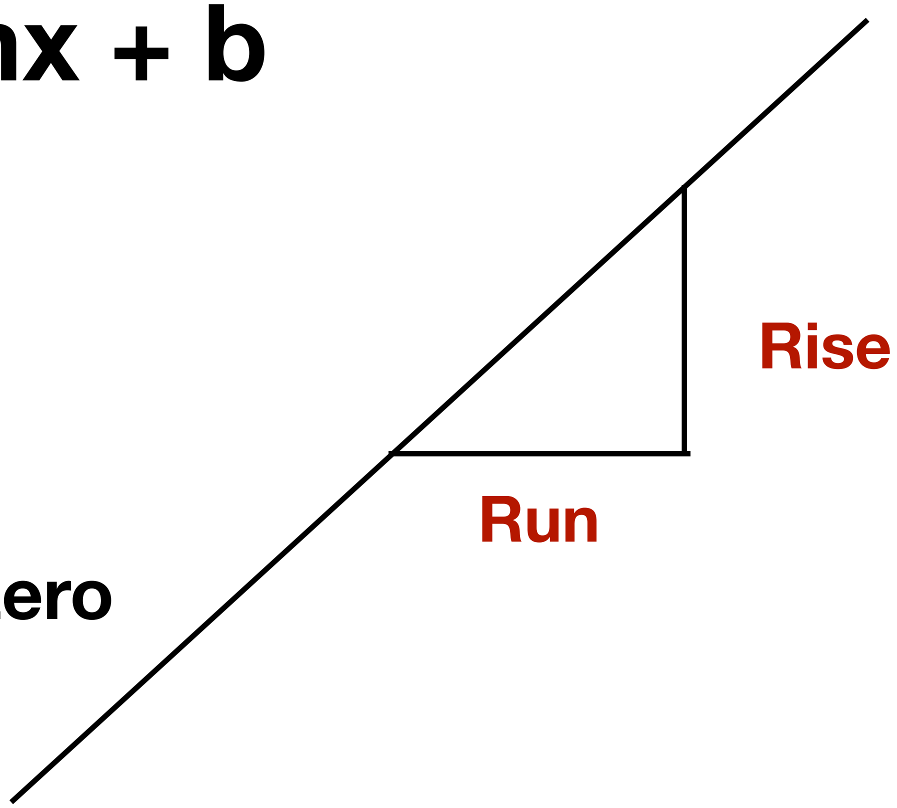
Coordinate system and Points



Points are written as:
(x-value, y-value)

Remember a line

$y = mx + b$

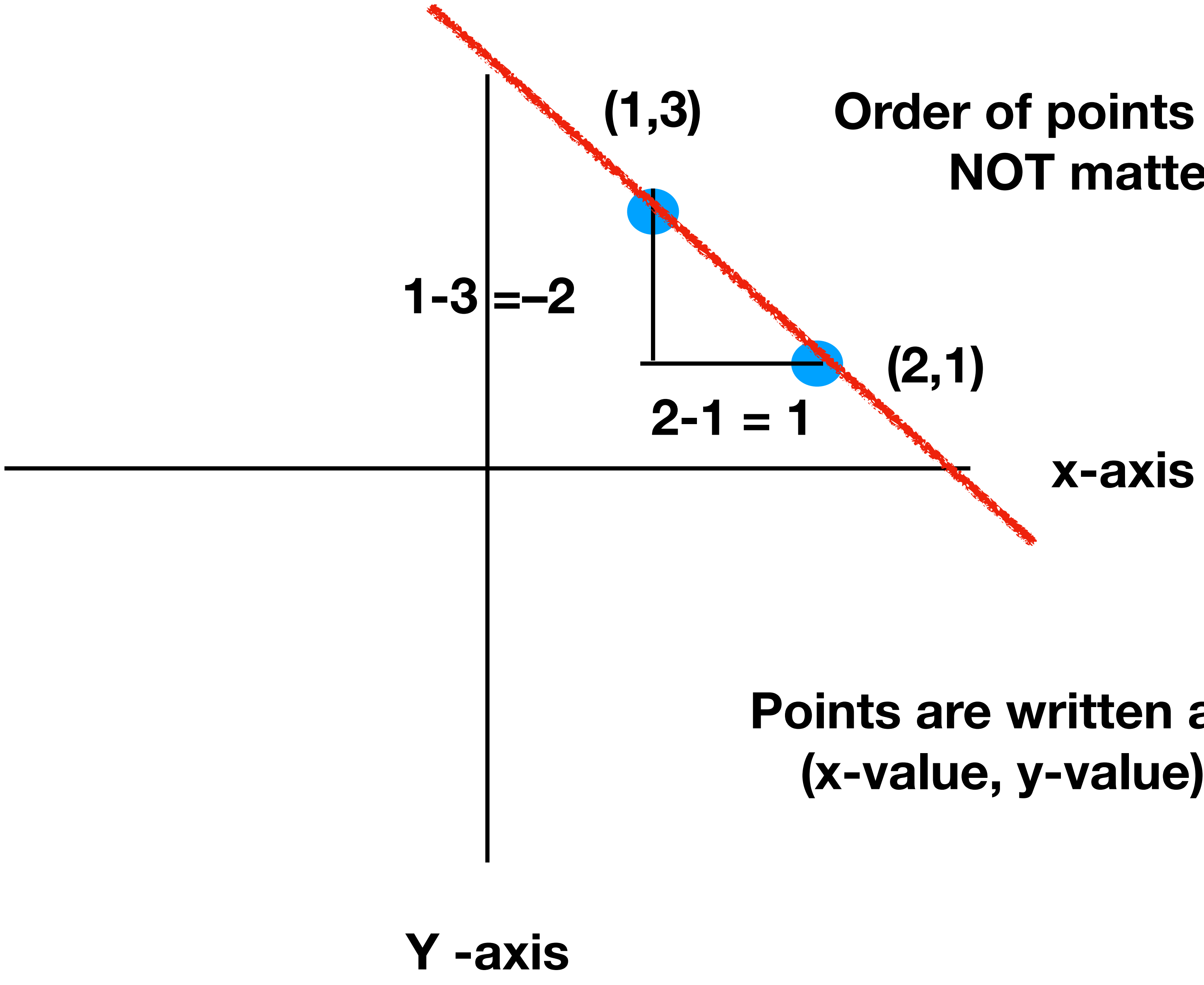


b -> intercept
or value of y when x is zero

m -> is the slope of the line
or “rise over run”

Finding Formula of line

Find the slope: -2/1



Order of points does NOT matter

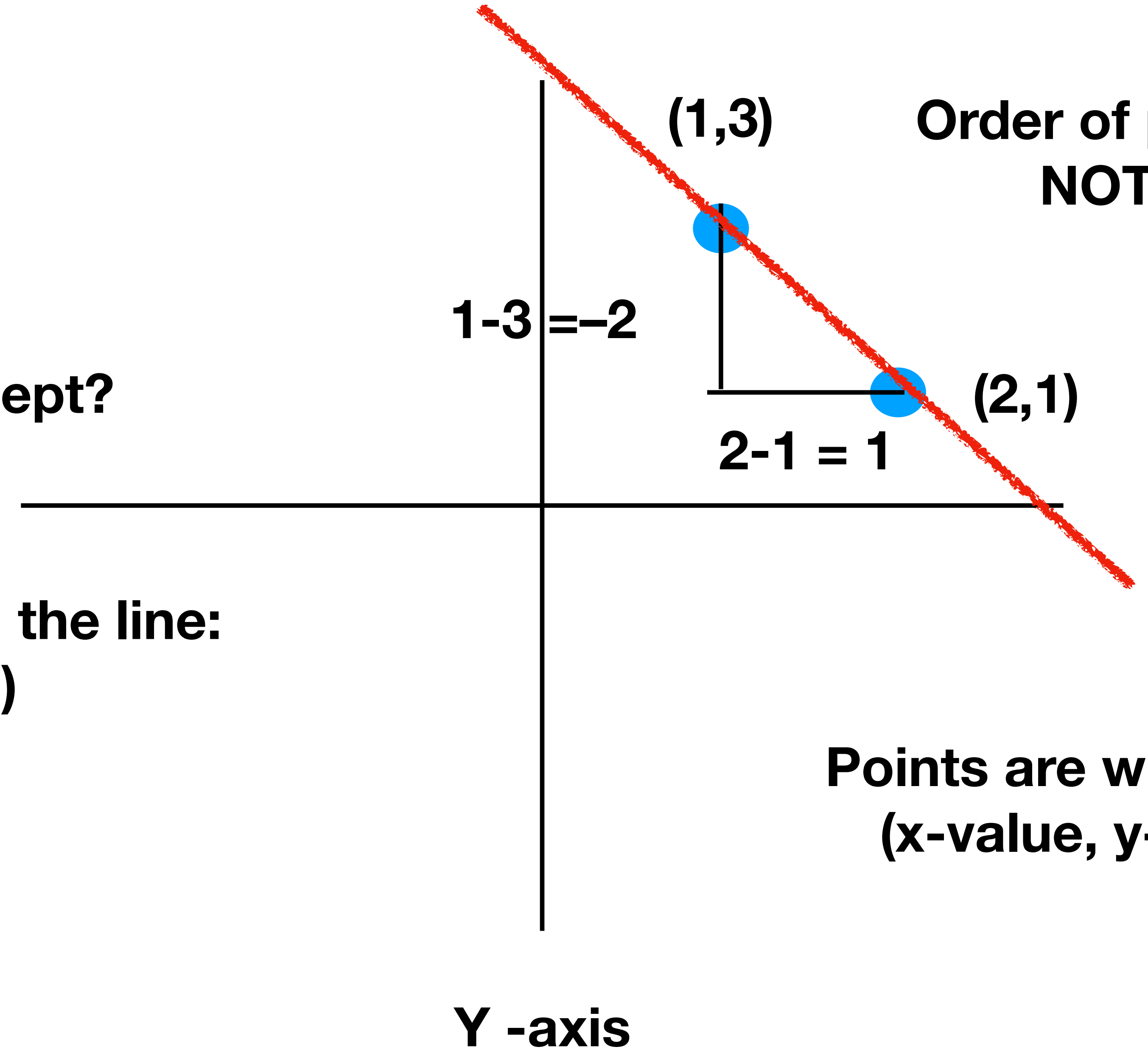
Points are written as:
(x-value, y-value)

Finding Formula of line

Find the slope: -2/1

What about the intercept?

Choose any point on the line:
Y - 3 = m(x-1)
Y-3 = -2(x-1)
Y-3 = -2x +1
Y = -2x +4



Order of points does NOT matter

Points are written as:
(x-value, y-value)

Scatter plots

- Often we are interested in the relationship between two variables

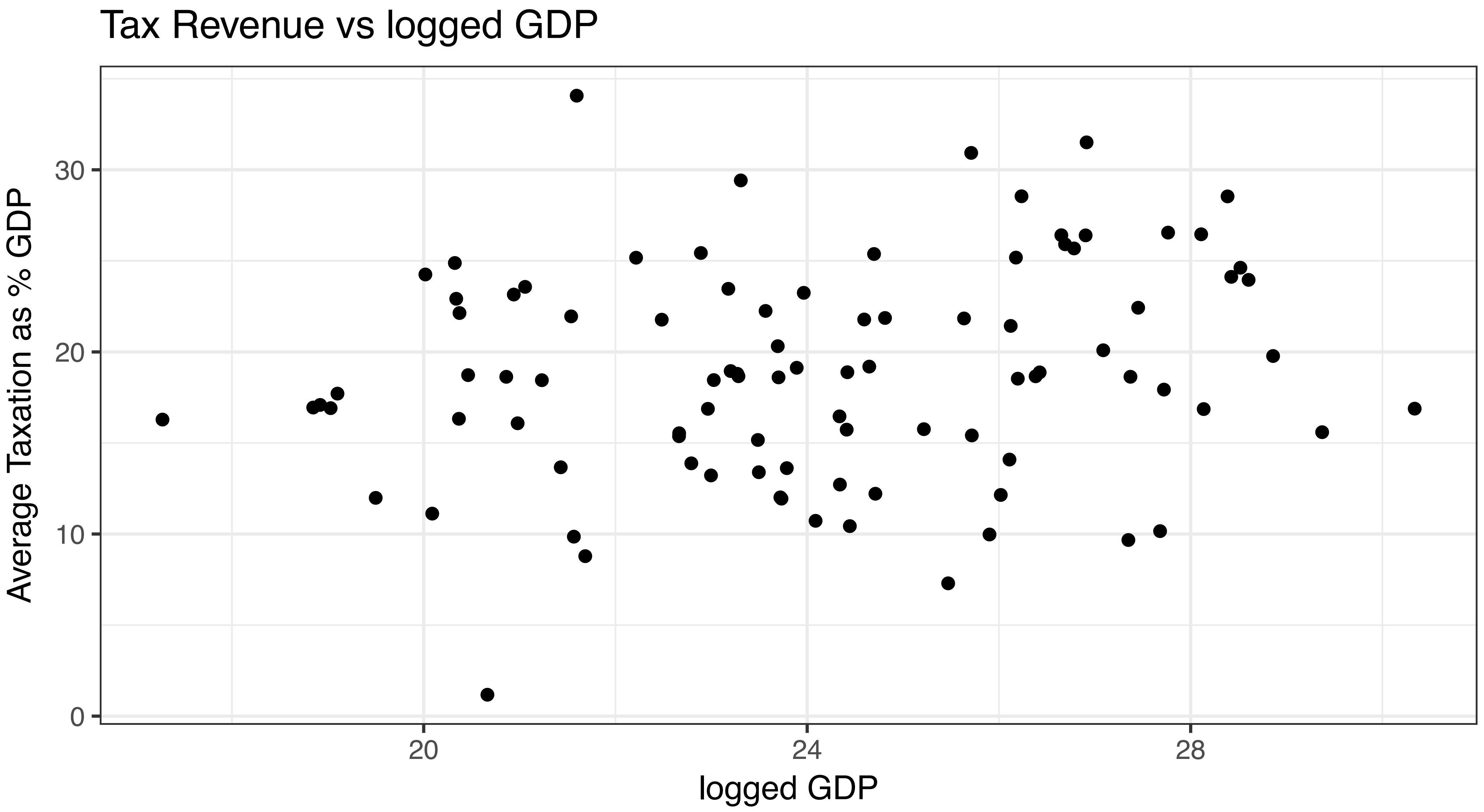
  A. Dependent variable: Y

  B. Independent variable: X

- Each observation has a y-value and x-value

Scatter plots

- We can plot each observation as a point on the coordinate system
- Each plot is drawn as if their y and x values are coordinates
- Scatter the points across the plot



Tax Revenue vs logged GDP
Average Taxation as % GDP
0
10
20
30
20
24
28
logged GDP

Correlation

- Correlation is a standardized measure of the co-relationship between two variables

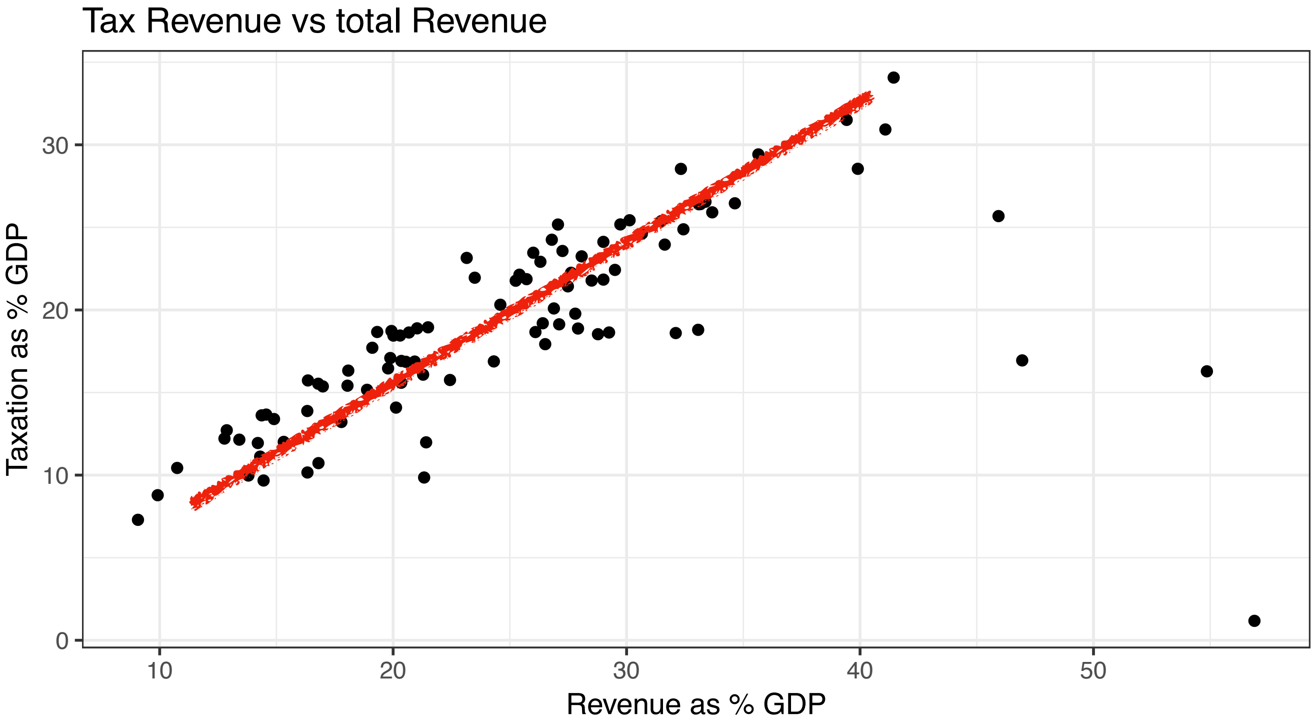
- Or, correlation is a standardized measure of the co-variation between two variables

- I.e how much do two variables move together?

Correlation

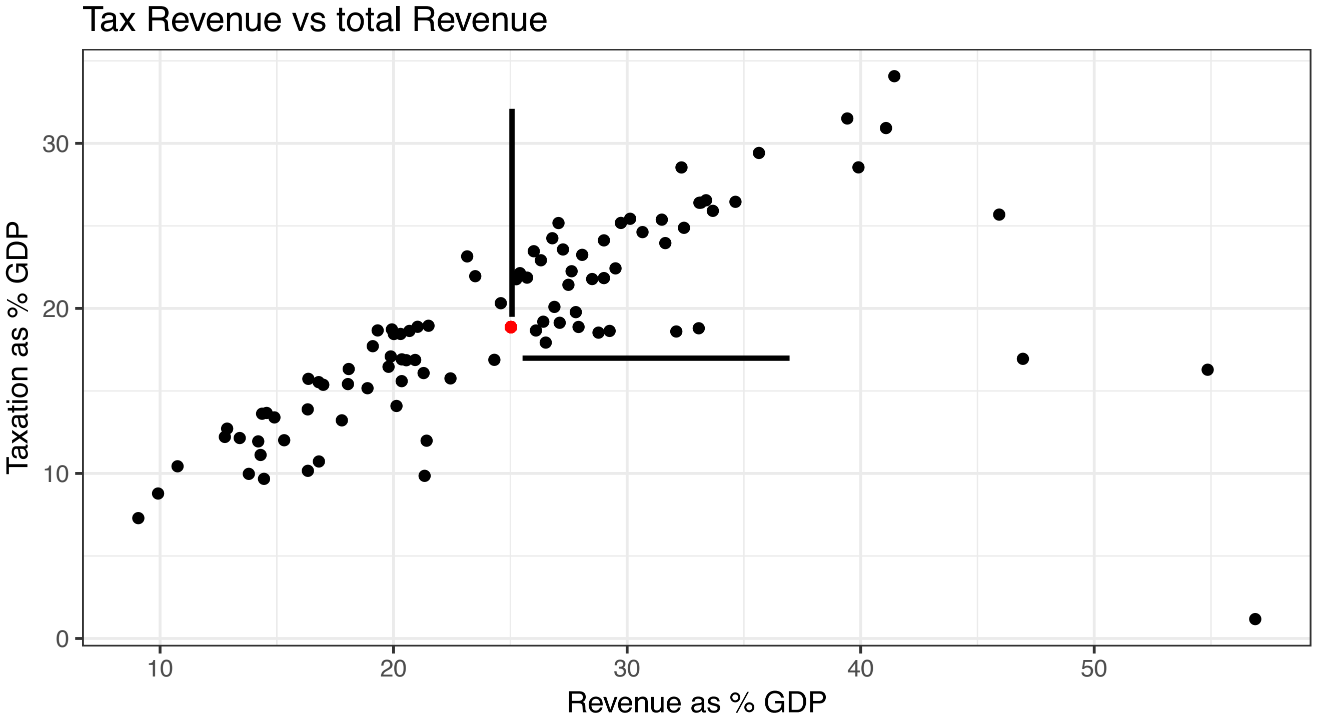
If there is a strong correlation (association) between two variables then knowing one will help predicting the other. If the correlation (association) is weak, then info about one helps little to predict the other.

Strong correlation looks like a very tight cloud of points that you could easily draw a line through



Remember, usually we call the variable we are interested in explaining or predicting the dependent variable (Y)

The spread of points in any direction is approximated by mean +/- 2*SD
Why?



y-mean = 18.87

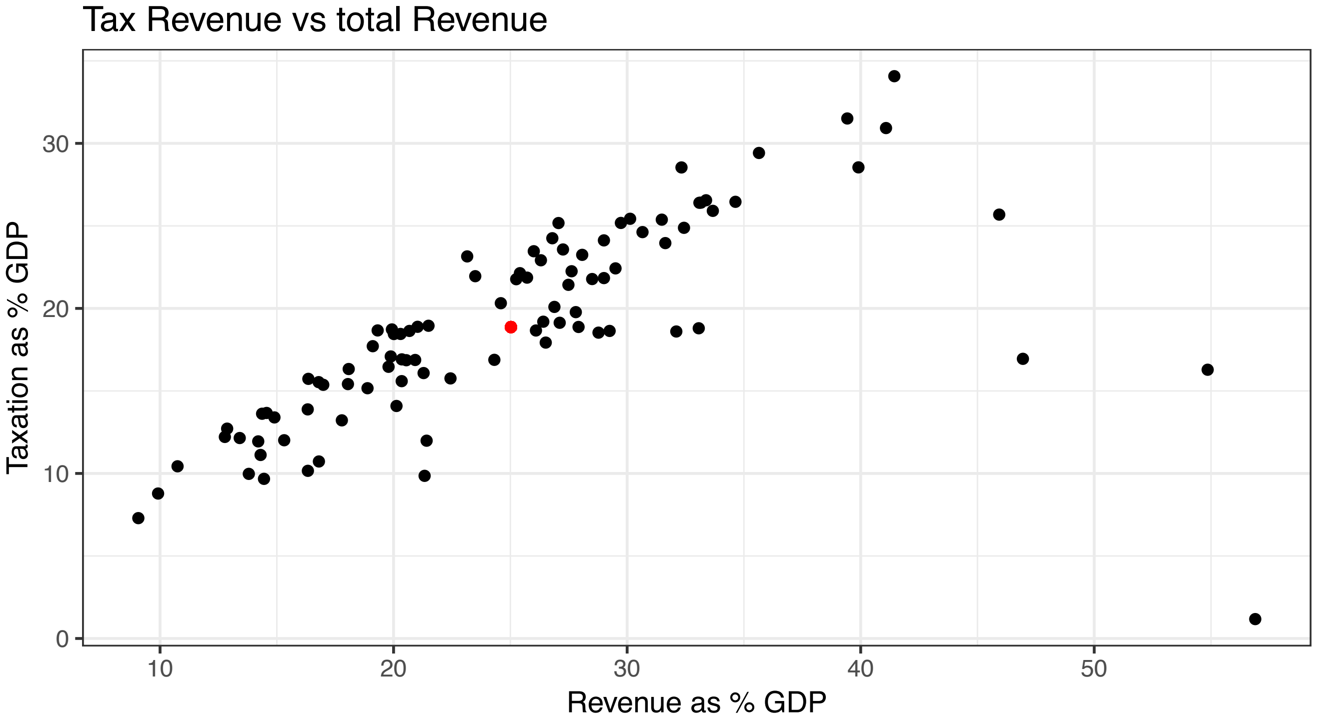
y-sd = 5.92

x-mean = 25.02

x-sd = 9.13

**x ranges approximately from 7 to 43,
y from 6 to 30**

The correlation is stronger the less spread out and steeper the imaginary cloud is



y-mean = 18.87

y-sd = 5.92

x-mean = 25.02

x-sd = 9.13

x ranges approximately from 7 to 43, y from 6 to 30

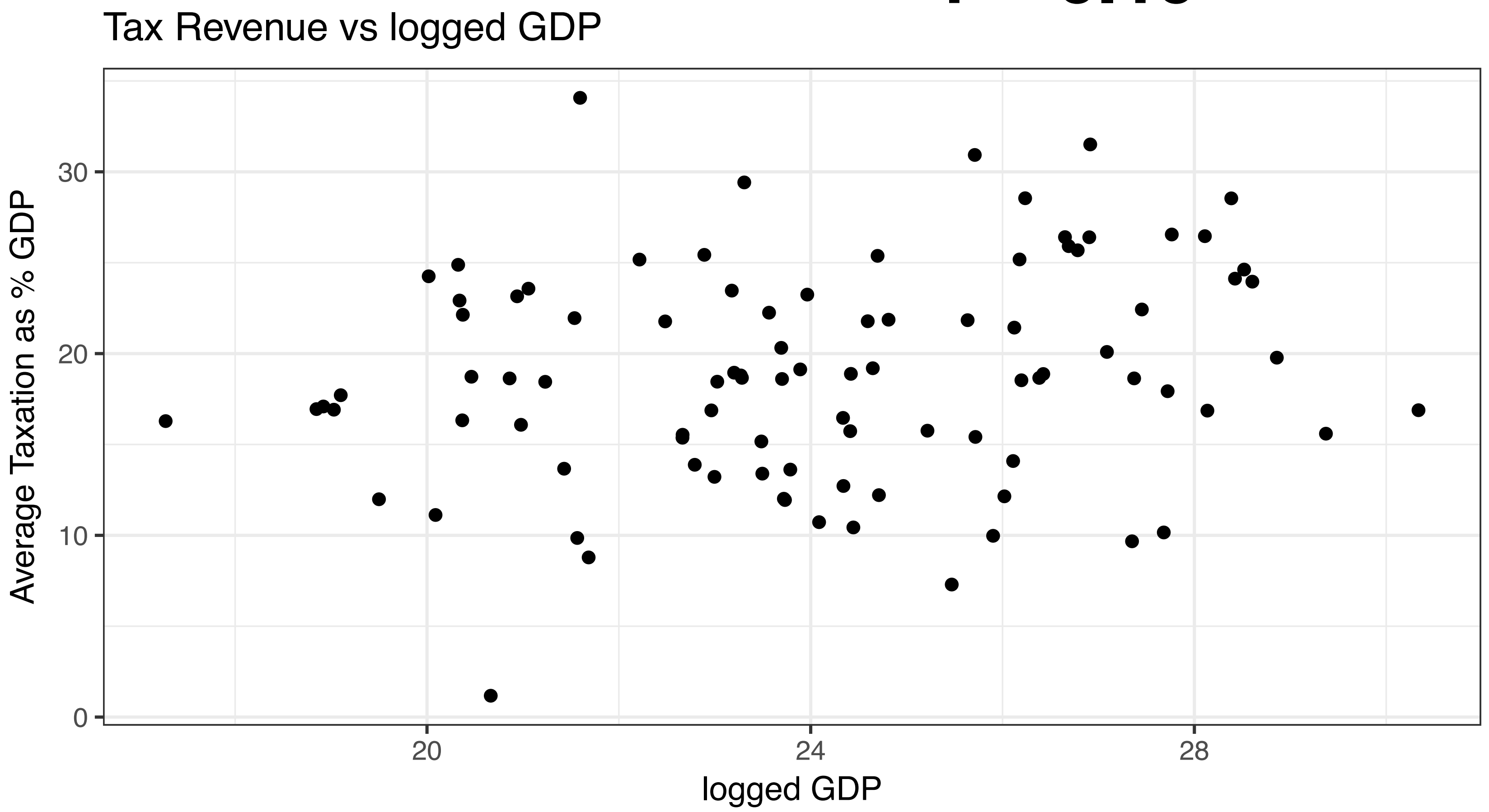
Correlation Coefficient r

- Ranges from -1 to 1
- -1 means strongest possible negative correlation
- 1 means strongest possible positive correlation
- 0 means no relationship

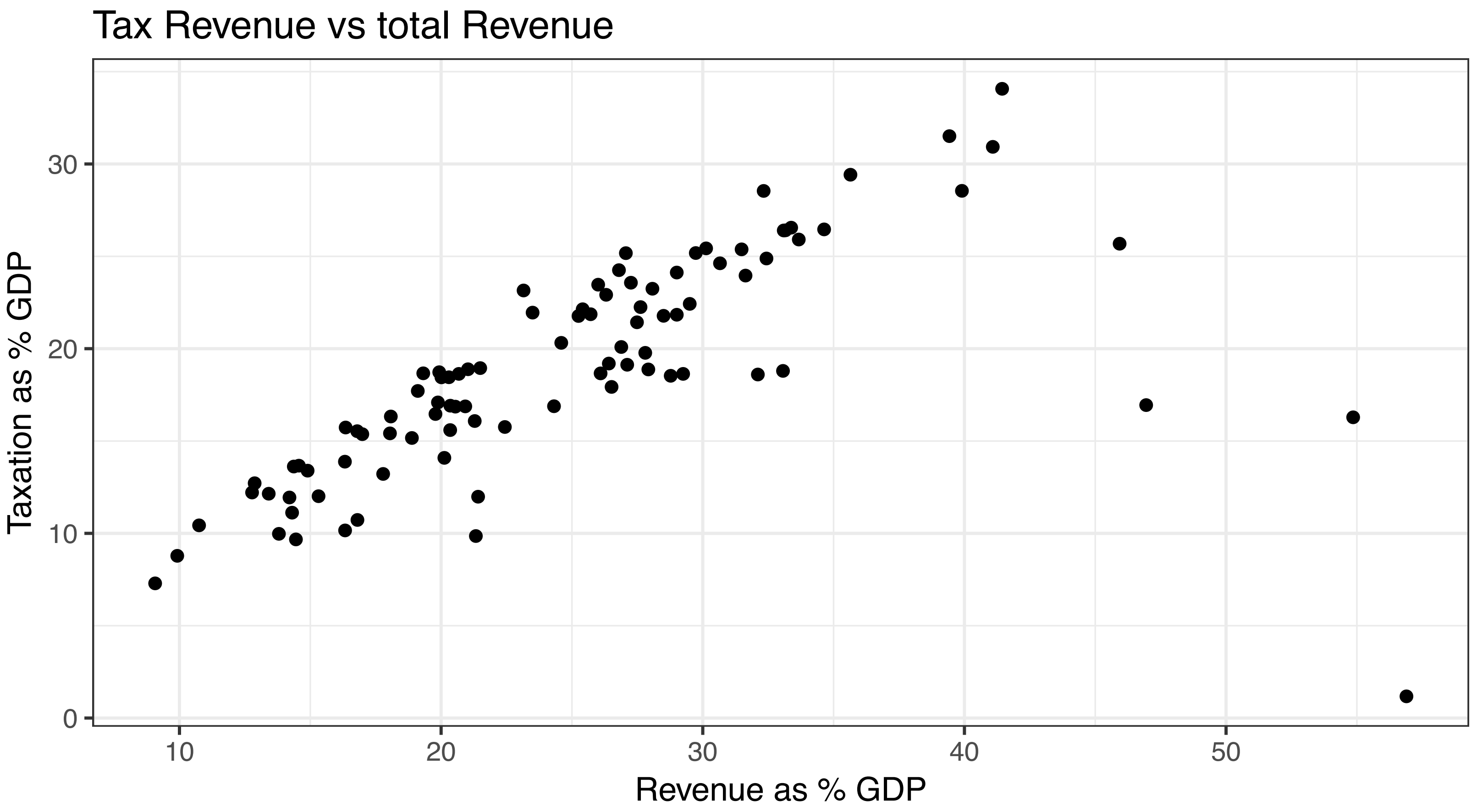
Correlation Coefficient r

- Negative value means slope is downwards
- Positive value means slope is upwards
- 0 means flat

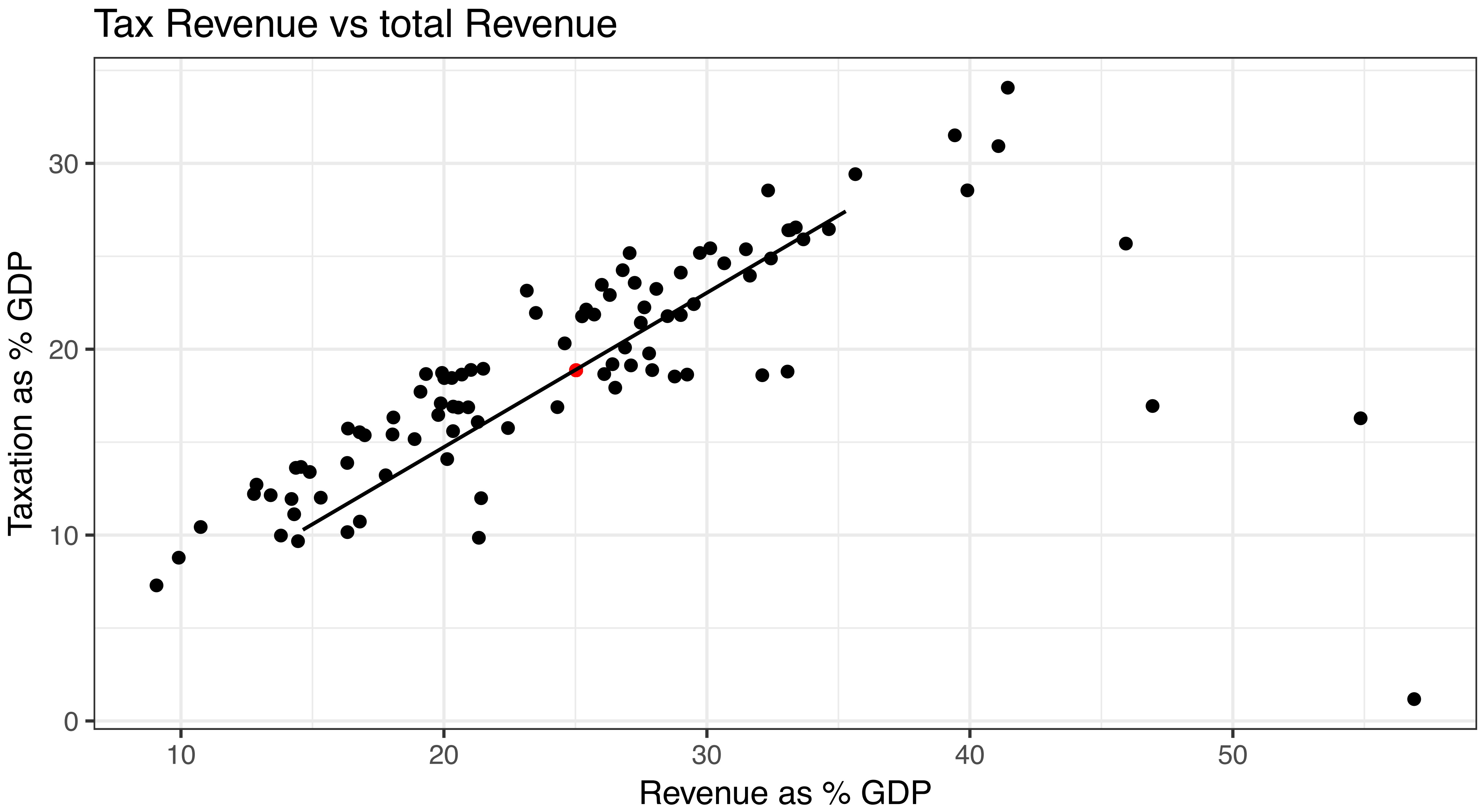
r = 0.19


Tax Revenue vs logged GDP
Average Taxation as % GDP
0
10
20
30
20
24
28
logged GDP

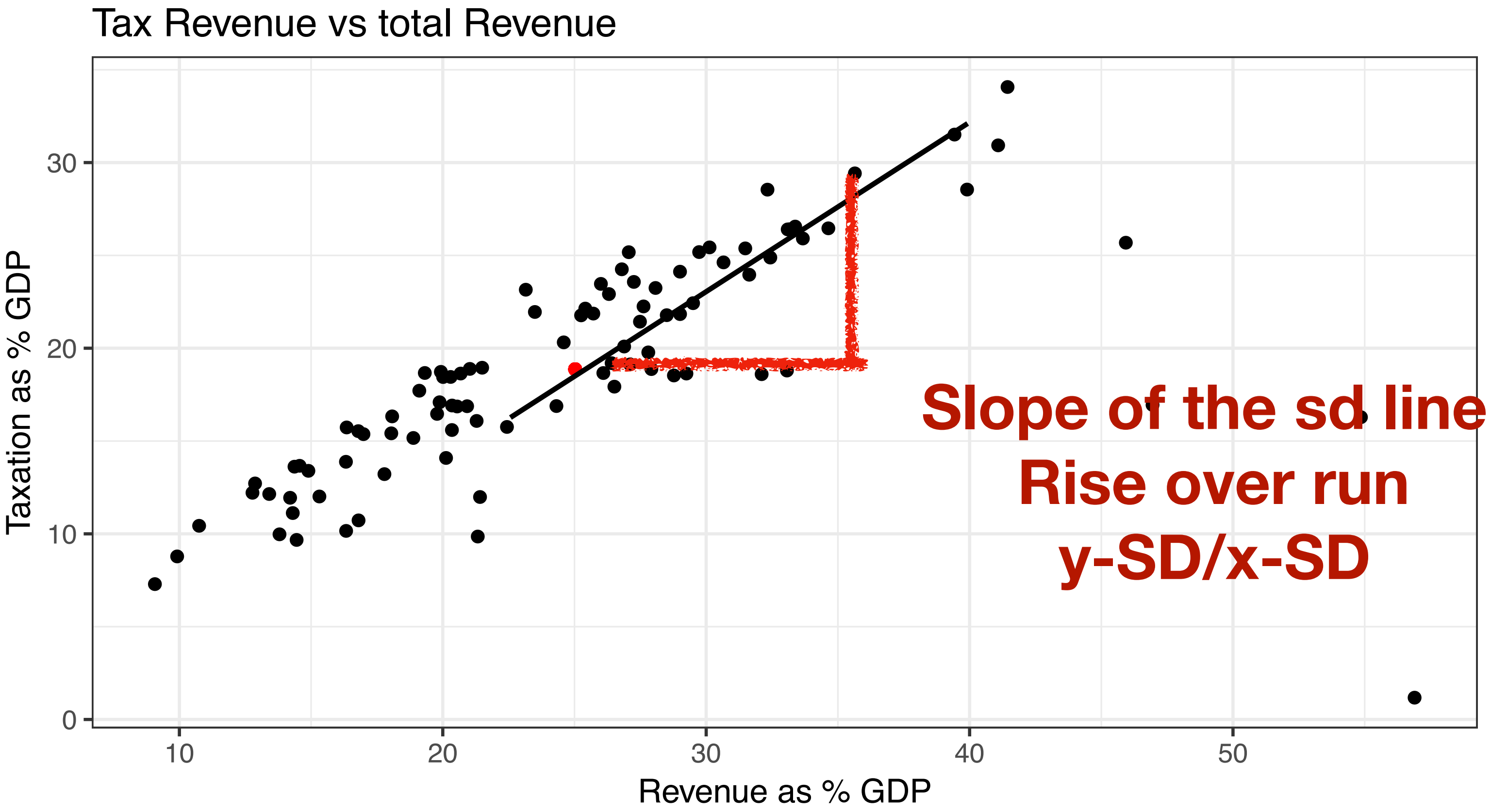
r = 0.57


Tax Revenue vs total Revenue
Taxation as % GDP
0
10
20
30
Revenue as % GDP
10
20
30
40
50

SD line goes through the point of averages and all points an equal number of SDs away from the average



SD line goes through the point of averages and all points an equal number of SDs away from the average



Slope of the sd line:
Rise over run
y-SD/x-SD

Calculating the correlation coefficient

- r = mean of (x in standard units * y in standard units)
  - For each x: subtract mean from value and divide by SD
  - For each y: subtract mean from value and divide by SD
  - Multiply standard values of both and then divide by N (number of observations)

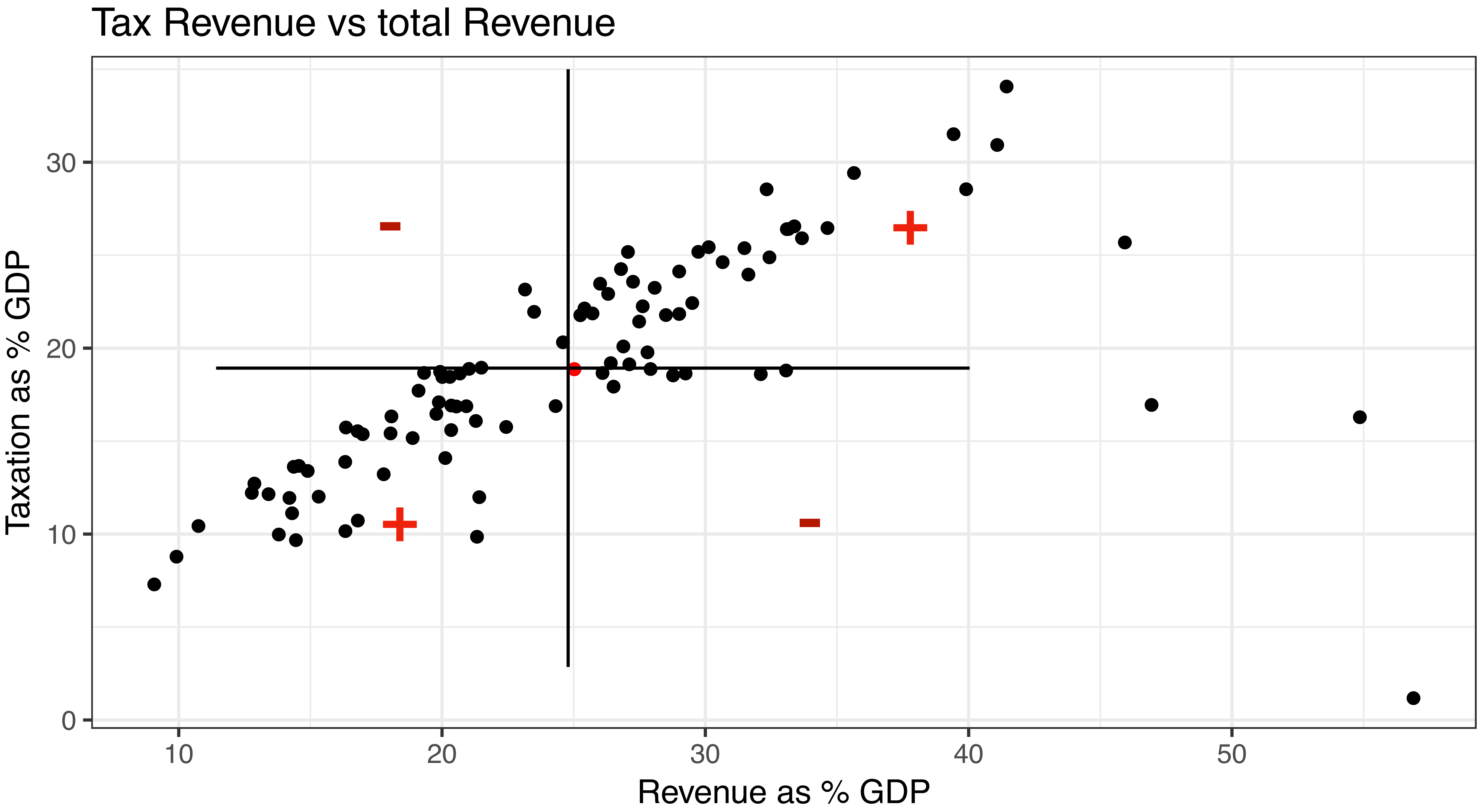
Let’s do an example

- Y = 1, 3, 4, 5, 7
- X = 5, 9, 7, 1, 13

Let’s do an example

- X = 1, 2, 3, 4, 5, 6, 7
- Y = 2, 1, 4, 3, 7, 5, 6

Why does the formula work?



If points in off-diagonal dominate, positive r
If points on diagonal dominate, negative r